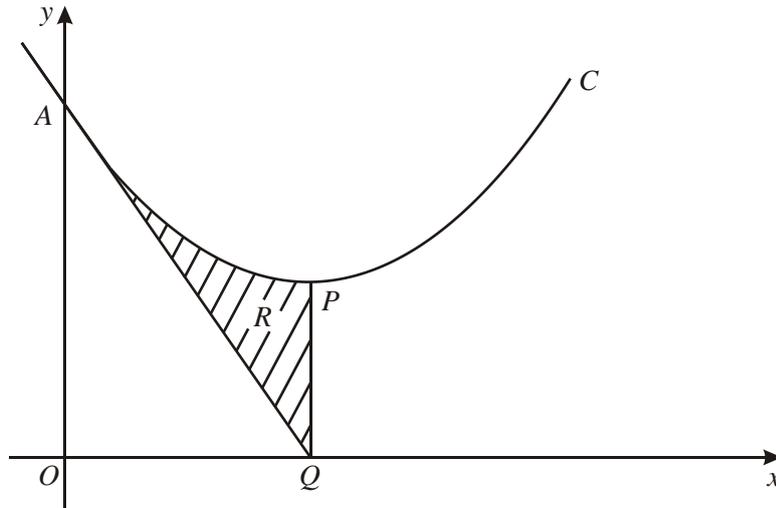**1.**

The diagram above shows part of the curve C with equation $y = x^2 - 6x + 18$. The curve meets the y-axis at the point A and has a minimum at the point P.

(a) Express $x^2 - 6x + 18$ in the form $(x - a)^2 + b$, where a and b are integers.

(3)

(b) Find the coordinates of P.
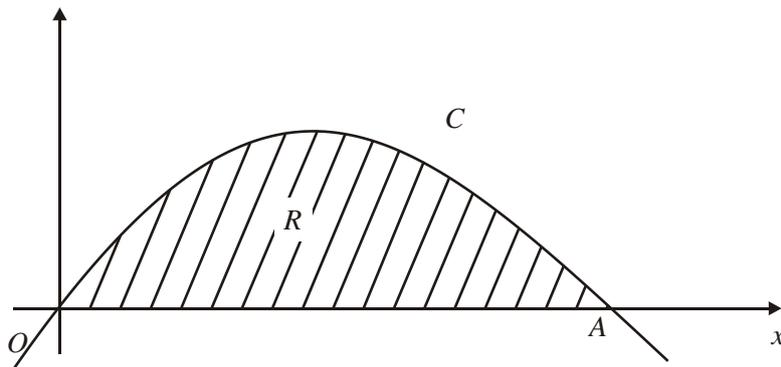
(2)

(c) Find an equation of the tangent to C at A.

(4)

The tangent to C at A meets the x-axis at the point Q.

(d) Verify that PQ is parallel to the y-axis.

(1)

The shaded region R in the diagram is enclosed by C, the tangent at A and the line PQ.

(e) Use calculus to find the area of R.

(5)
(Total 15 marks)

2.

The curve *C*, with equation $y = x(4 - x)$, intersects the *x*-axis at the origin *O* and at the point *A*, as shown in the diagram above. At the point *P* on *C* the gradient of the tangent is –2.
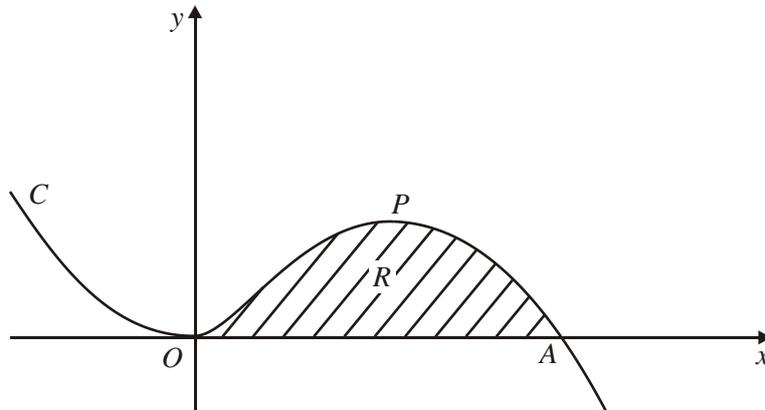
(a)   Find the coordinates of *P*.

(4)

The region *R*, enclosed between *C* and *OA*, is shown shaded.

(b)   Find the exact area of *R*.

(5)
(Total 9 marks)

3.

The diagram above shows part of the curve C with equation

$$y = \tfrac{3}{2} x^2 - \tfrac{1}{4} x^3.$$

The curve C touches the x-axis at the origin and passes through the point $A(p, 0)$.

(a)   Show that $p = 6$.

(1)

(b)   Find an equation of the tangent to C at A.

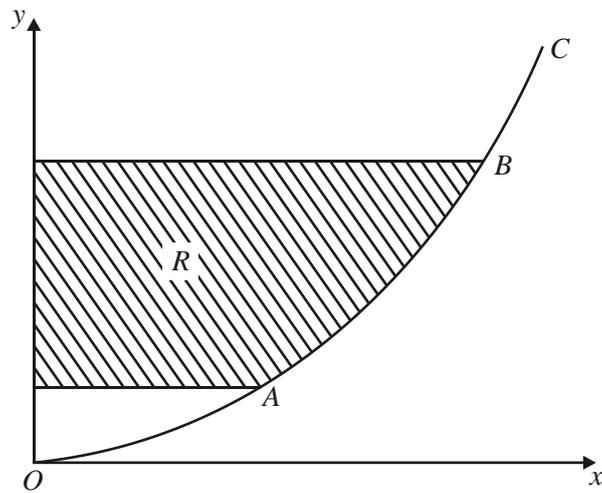**(4)**

The curve C has a maximum at the point P.

(c)   Find the x-coordinate of P.

(2)

The shaded region R, in the diagram above, is bounded by C and the x-axis.

(d)   Find the area of R.

(4)
(Total 11 marks)

4.

The curve *C*, shown in the diagram above, represents the graph of

$$y = \frac{x^2}{25}, \quad x \geq 0.$$

The points *A* and *B* on the curve *C* have *x*-coordinates 5 and 10 respectively.

(a)   Write down the *y*-coordinates of *A* and *B*.

(1)

(b)   Find an equation of the tangent to *C* at *A*.
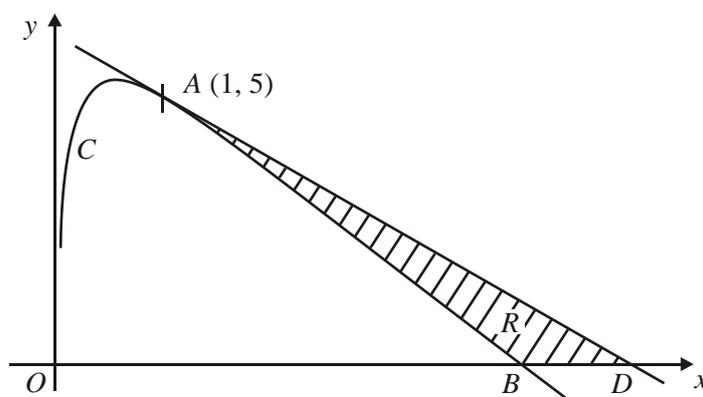
(4)

The finite region *R* is enclosed by *C*, the *y*-axis and the lines through *A* and *B* parallel to the *x*-axis.

(c)   For points (*x*, *y*) on *C*, express *x* in terms of *y*.

(2)

(d)   Use integration to find the area of *R*.

(5)
(Total 12 marks)

5.

The diagram above shows part of the curve C with equation

$$y = 9 - 2x - \frac{2}{\sqrt{x}}, \qquad x > 0.$$

The point $A(1, 5)$ lies on C and the curve crosses the x-axis at $B(b, 0)$, where b is a constant and $b > 0$.

(a)   Verify that $b = 4$.

(1)

The tangent to C at the point A cuts the x-axis at the point D, as shown in the diagram above.

(b)   Show that an equation of the tangent to C at A is $y + x = 6$.

(4)

(c)   Find the coordinates of the point D.

(1)

The shaded region R, shown in the diagram above, is bounded by C, the line AD and the x-axis.

(d)   Use integration to find the area of R.

(6)
(Total 12 marks)

1. (a) $(x-3)^2$, $+9$ isw . $a=3$ and $b=9$ may just be written
        down with no method shown.   B1, M1 A1   3

    (b) P is (3, 9)   B1

    (c) A = (0, 18)   B1

        $\dfrac{dy}{dx} = 2x-6$, at A $m=-6$ M1 A1

        Equation of tangent is $y-18 = -6x$ (in any form)   A1ft   4

    (d) Showing that line meets x axis directly below P, i.e. at $x=3$.   A1cso   1

    (e) $A = \int x^2 - 6x + 18x = [\dfrac{1}{3}x^3 - 3x^2 + 18x]$ M1 A1

        Substituting $x=3$ to find area A under curve A [=36]   M1

        Area of $R = A -$ area of triangle $= A - \dfrac{1}{2} \times 18\ 3, = 9$ M1 A1   5

        Alternative: $\int x^2 - 6x + 18 - (18 - 6x)dx$ M1
                     $= \frac{1}{3}x^3$ M1 A1 ft
                Use $x=3$ to give answer 9   M1 A1

                                                                       [13]

2. (a) $y = 4x - x^2$ $\dfrac{dy}{dx} = 4 - 2x$ M1 A1

        "$4 - 2x$" $= -2$, $x = …$ M1
        $x = 3$, $y = 3$ A1   4

    (b) x-coordinate of A is 4   B1

        $\int (4x - x^2)dx = \left[\dfrac{4x^2}{2} - \dfrac{x^3}{3} \right]$ M1 A1ft

        $\left[\dfrac{4x^2}{2} - \dfrac{x^3}{3} \right]_0^4 = 32 - \dfrac{64}{3} = \dfrac{32}{3}$ $\left(= 10\dfrac{2}{3} \right)$ (or exact equivalent)   M1 A1   5

                                                                       [9]

3. (a) Solve $\dfrac{3}{2}x^2 - \dfrac{1}{4}x^3 = 0$ to find $p = 6$, or verify: $\dfrac{3}{2} \times 6^2 - \dfrac{1}{4} \times 6^3 = 0$ (*)   B1   1

    (b) $\dfrac{dy}{dx} = 3x - \dfrac{3x^2}{4}$ M1 A1

        $m = -9$, $y - 0 = -9(x - 6)$ (Any correct form)   M1 A1   4

(c) $3x - \dfrac{3x^2}{4} = 0$, $x = 4$ M1, A1ft   2

(d) $\displaystyle\int\left(\dfrac{3x^2}{2} - \dfrac{x^3}{4}\right)dx = \dfrac{x^3}{2} - \dfrac{x^4}{16}$ (Allow unsimplified versions)   M1 A1

$[.........]_0^6 = \dfrac{6^3}{2} - \dfrac{6^4}{16} = 27$ M: Need 6 and 0 as limits.   M1 A1   4

[11]

4.   (a) $A: y = 1$ $B: y = 4$ B1

(b) $\dfrac{d\,y}{d\,x} = \dfrac{2x}{25}$ $= \dfrac{2}{5}$ where $x = 5$ M1 A1

Tangent: $y - 1 = \dfrac{2}{5}(x - 5)$ $(5y = 2x - 5)$ M1 A1

(c) $x = 5y^{\frac{1}{2}}$ B1 B1

(d)   Integrate: $\dfrac{5y^{\frac{3}{2}}}{\frac{3}{2}}$ $\left(= \dfrac{10y^{\frac{3}{2}}}{3}\right)$ M1 A1 ft

$[\,]^4 - [\,]_1 = \left(\dfrac{10 \times 4^{\frac{3}{2}}}{3}\right) - \left(\dfrac{10 \times 1^{\frac{3}{2}}}{3}\right), = \dfrac{70}{3}$ $(23\frac{1}{3}, 23.3)$ M1 A1, A1

Alternative for (d):   Integrate: $\dfrac{x^3}{75}$ M1 A1

Area $= (10 \times 4) - (5 \times 1) - \left(\dfrac{1000}{75} - \dfrac{125}{75}\right), = \dfrac{70}{3}$ $(23\frac{1}{3}, 23.3)$ M1 A1, A1

In both (d) schemes, final M is scored using <u>candidate's</u> "4" and "1".

[12]

5.   (a) $y = 9 - 8 - \dfrac{2}{\sqrt{4}} = 0 \therefore b = 4$ **(*)**   B1 c.s.o.   1

(b) $\dfrac{dy}{dx} = -2 + x^{-\frac{3}{2}}$ M1

When $x = 1$ gradient $= -2 + 1 = -1$ A1

So equation of the tangent is $y - 5 = -1\,(x - 1)$ M1

i.e. $y + x = 6$ **(*)**   A1 c.s.o.   4

(c)   Let $y = 0$ and $x = 6$ so D is $(6, 0)$ B1   1

(d)   Area of triangle $= \frac{1}{2} \times 5 \times 5 = 12.5$ B1

$$\int_1^4 (9 - 2x - 2x^{-\frac{1}{2}})\,dx = \left[9x - x^2 - 2\frac{x^{\frac{1}{2}}}{\frac{1}{2}} \right]_1^4$$

Ignore limits   M1 A1

$= (36 - 16 - 4 \times 2) - (9 - 1 - 4)$

Use of limits   M1

$= 12 - 4$

$= 8$ A1

So shaded area is $12.5 - 8 = 4.5$ A1   6

[12]

1.   (a)   This was generally answered very well. Many candidates scored full marks, with others gaining B1, M1, A0, for answers such as $(x-3)^2$-9, or -27, or +27

(b)   The majority of candidates did not use their (a) to get the answer in (b) but used differentiation. This meant that most candidates had this part correct, even if they had (a) incorrect or didn't attempt it.

(c)   Again this part was well answered, especially by those who had done the differentiation in (b) as they went on to get the gradient of -6, and used (0,18) to get the equation of the line. Unfortunately, some assumed the coordinates of Q at (3,0), and found the gradient using the points A and Q, so didn't gain many marks at all in this part.

(d)   Generally if candidates had answered part (c) correctly, they were able to do part (d) as well, although quite a few lost credibility because they stated that the gradient was 0. Many compared the x coordinates and deduced the line was parallel to the y axis and gained the credit.

(e)   This was very well answered. Many candidates had full marks in this part even if they hadn't scored full marks earlier. A few candidates made the mistake of using the y value of 9 instead of the x value of 3 in the integral. Other errors included using a trapezium instead of a triangle, and some candidates made small slips such as the 18 being copied down as an 8 or integrating the 6x to get 6x/2, or $6x^2$. It is possible that these candidates were short of time.

2.   In part (a) of this question, some candidates failed to appreciate the need to differentiate and therefore made little progress. Such candidates often tried, in various ways, to use an equation of a straight line with gradient –2, perhaps inappropriately passing through the point *A*. Those who did differentiate were often successful, although some ignored the –2 and equated their derivative to zero.

There were many completely correct solutions to part (b). Integration techniques were usually sound, but a few candidates had difficulty in finding the correct limits, especially where they looked for a link between parts (a) and (b).

3.   Although there were many very good solutions to this question, a large number of candidates failed to cope with the parts requiring applications of differentiation.

Most scored the mark in part (a), either by solving an equation or by verification, for showing that $p = 6$, although arguments were occasionally incomplete.

Differentiation was required to answer parts (b) and (c) and most candidates scored the two marks for a correct derivative, seen in either of these parts. A significant number omitted part (b). Some found the gradient at *A* but did not proceed to find the equation of the tangent, some found the gradient of the normal, and some gave a non-linear tangent equation, failing to evaluate the derivative at $x = 6$. There was rather more success in part (c), although a common mistake here was to equate the second derivative to zero in the attempt to find the maximum turning point.

In part (d), most candidates appreciated the need for definite integration, and many completely correct solutions were seen. A few, however, used $x = 4$ (presumably taken from part (c)) instead of $x = 6$ as the upper integral limit.

4.   Although many candidates found this question difficult, those who were competent in algebra and calculus often produced excellent, concise solutions.

Part (a) caused very little difficulty, almost everyone scoring the one available mark. In part (b), however, attempts at finding the equation of the tangent were disappointing. Some candidates made no attempt to differentiate, while others differentiated correctly to get $\dfrac{2x}{25}$, but failed to find the gradient at $x = 5$, using instead $\dfrac{2}{25}$, or even $\dfrac{2x}{25}$, as the gradient *m* in the equation of the tangent.

Although some had no idea what to do in part (c), the majority were able to express *x* as $\sqrt{25y}$ or equivalent. Here, it was not always clear whether the square root extended to the *y* as well as the 25, but a correct expression in part (d) gained the marks retrospectively.

The expression of *x* in terms of *y* was, of course, intended as a hint for part (d), but many candidates happily ignored this and integrated *y* with respect to *x* rather than *x* with respect to *y*. Using this approach, just a few were able to find the area of the required region by a process of subtracting the appropriate areas from a rectangle of area 40, but it was very rare to see a complete method here. Some candidates wrongly used 5 and 10 as *y* limits (or 1 and 4 as *x* limits). Those integrating 5Öy with respect to *y* were very often successful in reaching the correct answer.

5. No Report available for this question.